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"VARIATIONAL PRINCIPLE FOR REYNOLD'S NUMBER IN CASE OF STRATIFIED FLUID"

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Abstract

Work done in this paper guides us to find out the relation involving Reynold's number for modes of non-oscillatory nature using calculus of variations under the assumption that fluid under discussion is having different layers passing through matter having pores. These relations help in stability problems, water waves propagation, rarefied gas flow problems and optimizing thrust from exhaust nozzles. While deriving the relations, we consider the moving fluid under the impact of heat and magnetizing force.

Keywords: *Chandrasekhar number, eigen value, pressure gradient, oscillatory modes, perturbation*

Some important concepts: -

Density: It is the mass contained in one unit volume.
Pressure: Pressure is Force acting on unit area of body.
Pressure Gradient: We define Pressure as Force acting on unit area of body. So, if there is difference in force across any surface then correspondingly there is difference in pressure across the surface and hence the concept of pressure gradient arises.
Normal mode: A type of motion involving a pattern of movement of constituents of whole body having equal frequency along with a definite phase relationship.
Compressible and incompressible fluids: If the volume and hence density of a fluid changes then it is called compressible fluid whereas, an incompressible fluid is one which can neither be compressed nor expanded, and its volume and therefore its density is always constant.
Porous object: A matter made up of vacant spaces in between enabling a fluid easily pass through.
Grashof number: Quotient of buoyancy force and force due to viscous characteristics of fluid moving with some velocity.

Rayleigh number: The Grashof number multiplied with the Prandtl number gives Rayleigh number**.**
Stratified fluid flow: A flow of fluids is called stratified if the lightest fluid flows in the top layer accompanied by heavier fluid and the heaviest fluid flowing at the bottom of the flow.

Introduction

Heat propagation in a fluid made up of varying constituents and flowing through a porous matter has a substantial and pronounced significance. This concept is of utmost and sizeable value while studying nature of soil, study of earth's surface, ground water hydrology, during pumping out oil from under the surface and nuclear processes. Khare and Sahai in 1993 studied the impact of temperature on motion of heterogenous fluid flowing through a porous matter impacted by magnetizing field. Khare and Sahai in 1993 also gave result on instability property shown by such fluid using calculus of variations. Other authors have also discussed the behavior of such fluids ([1]-[4]).

Let us discuss the problem:

We assume a wet porous substance placed horizontally. Let the thickness of the substance is $l\,(l>0)$ .Suppose it is placed in between two unbounded parallel planes P_1'' and Q_1'' lying vertically one above the other at height 0 and l respectively. Let the lower plane is having temperature $t = t_0$ and upper plane is having temperature $t = t_1\,(t_0 > t_1)$.The fluid under discussion is thick, having constant density while it is flowing. Under this assumption, $d\Omega/dx$ remains constant. We also take into consideration that a magnetizing force is acting vertically to our setup[1], then, we can write the equations of motion (Khare and Sahai,1993) as:

$$\rho_0'\left[\frac{\partial u_i'}{\partial t'} + u_j'\frac{\partial u_i'}{\partial x_j'}\right] = -\frac{\partial P'}{\partial x_i'} + \frac{\mu_e'}{4\pi}H_j'\frac{\partial H_i'}{\partial x_j'} - \frac{\mu_e'}{8\pi}\frac{\partial}{\partial x_i'}(H_j' H_j') + \rho' x_i' - \rho_0'\frac{v'}{k'}u_i' + \rho_0' v' \nabla^2 u_i'$$

$$\frac{\partial \rho'}{\partial t'} + u_j'\frac{\partial \rho'}{\partial x_j'} = 0 \ ,$$

$$\frac{\partial T'}{\partial t'} + u_j'\frac{\partial T'}{\partial x_j'} = k_T'\nabla^2 T'$$

$$\frac{\partial H_i'}{\partial t'} + u_j'\frac{\partial H_i'}{\partial x_j'} = H_j'\frac{\partial u_i'}{\partial x_j'} + \eta'\nabla^2 H_i' \ ,$$

$$\nabla . u' = 0,$$

$$\nabla . H' = 0$$

$\rho' = \rho_0'[\Omega(x) + \alpha'(t_0 - t')]$, where x_1', x_2', x_3' denote, respectively, the co-ordinates along the axes, u_1', u_2', u_3' denote respectively the three components u', v', w' of velocity. X_1', X_2', X_3' are the components of the applied force and H_1', H_2', H_3' are the components of applied magnetizing force respectively(Khare and Sahai,1993). K_T' and K' symbolize thermal diffusivity and permeability of porous matter, $v' = \dfrac{\mu'}{\rho_0'}$ is the viscosity coefficient, α' is the coefficient of volume expansion and μ_e' is the magnetic permeability.

Now we are going to examine the stability of the unperturbed state which is presented by the following initial values for velocity, temperature, density and pressure gradient respectively,

$u_1' = 0, u_2' = 0, u_3' = 0$, $t' = t_1 - \wedge x$, $P' = -\int g\rho' \, dx$

where, $\wedge = (t_0 - t_1)/l$ is the ratio representing rate and direction of flow of temperature.

$H_1' = 0, H_2' = 0, H_3' = H'$

Suppose a small perturbation is induced in the initial state, therefore the perturbation state is characterized by,

$\overline{u_j'} = (u', v', w')$, $\overline{t_1}' = t_1 + \theta'$, $\overline{P}' = P' + \delta P'$

$$\overline{\rho}' = \rho_0' \left[\Omega(x) + \frac{\delta\rho'}{\rho_0'} + \alpha' (t_1 - t' - \theta') \right]$$

$\overline{H_1'} = h_x', \overline{H_2'} = h_y', \overline{H_3'} = H' + h_Z'$, where $u', v', w', \theta', \delta P', \delta\rho'$ and h_x', h_y', h_z' symbolize the perturbation induced velocity, temperature as result of disturbance, induced pressure, density of fluid and the magnetizing force[2].Then the perturbation equations in linear form can be written as:

$$\rho_0' \frac{\partial u'}{\partial t} = -\frac{\partial}{\partial x}\delta P_1' + \frac{\mu_e'}{4\pi} H' \frac{\partial h_x'}{\partial z} - \rho_0' \frac{\nu'}{k'} u' + \rho_0' \upsilon' \nabla^2 u' \quad (1)$$

$$\rho_0' \frac{\partial v'}{\partial t} = -\frac{\partial}{\partial y}\delta P_1' + \frac{\mu_e'}{4\pi} H' \frac{\partial h_y'}{\partial z} - \rho_0' \frac{\nu'}{k'} v' + \rho_0' \upsilon' \nabla^2 v' \quad (2)$$

$$\rho_0' \frac{\partial w'}{\partial t} = -\frac{\partial}{\partial z}\delta P'_1 + \frac{\mu_e'}{4\pi} H' \frac{\partial h_z'}{\partial z} - \rho_0' \frac{\nu'}{k'} w' + \rho_0' \upsilon' \nabla^2 w' - g\delta\rho' + g\alpha' \rho_0' \theta' \quad (3)$$

$$\frac{\partial(\delta\rho')}{\partial t} + \rho_0' w' \frac{d\Omega}{dx} = 0 \quad (4)$$

$$\frac{\partial\theta'}{\partial t} - \wedge w' = K_T' \nabla^2 \theta' \quad (5)$$

$$\frac{\partial h_x'}{\partial t} = H' \frac{\partial u'}{\partial z} + \eta' \nabla^2 h_x' \quad (6)$$

$$\frac{\partial h_y'}{\partial t} = H' \frac{\partial v'}{\partial z} + \eta' \nabla^2 h_y' \quad (7)$$

$$\frac{\partial h_z'}{\partial t} = H' \frac{\partial w'}{\partial z} + \eta' \nabla^2 h_z' \quad (8)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \quad (9)$$

$$\frac{\partial h_x'}{\partial x} + \frac{\partial h_y'}{\partial y} + \frac{\partial h_z'}{\partial z} = 0 \quad (10)$$

where,

$$\delta P_1' = \delta P' + \frac{\mu_e'}{4\pi} H' h_x'$$

We now investigate the quantities which translate the disturbances as normal modes and interpret their space and time dependence in the form below (Khare and Sahai,1993)

$$\Omega(x) e^{[i(k_x' X' + k_y' Y') + mt]} \quad (11)$$

where, $K^{'} = \sqrt{K_x^{2^{'}} + K_y^{2^{'}}}$ symbolizes the wave number of waves generated after inducing disturbances and m denotes number of complete oscillations per second during any arbitrary perturbation given. Then equations from (1) to (11) become

$$\rho_0^{'} m u^{'} = -iK_x^{'} \delta P_1^{'} + \frac{\mu_e^{'}}{4\pi} H^{'} \frac{d}{dx} h_x^{'} - \rho_0^{'} \frac{v^{'}}{k^{'}} u^{'} + \rho_0^{'} v^{'} (\left[\frac{d}{dx}\right]^2 - K^{'2}) u^{'} \quad (12)$$

$$\rho_0^{'} m v^{'} = -iK_y^{'} \delta P_1^{'} + \frac{\mu_e^{'}}{4\pi} H^{'} \frac{d}{dx} h_y^{'} - \rho_0^{'} \frac{v^{'}}{k^{'}} v^{'} + \rho_0^{'} v^{'} (\left[\frac{d}{dx}\right]^2 - K^{'2}) v^{'} \quad (13)$$

$$\rho_0^{'} n^{'} w^{'} = -\Delta \delta^{'} P_1^{'} + \frac{\mu_e^{'}}{4\pi} H^{'} \frac{d}{dx} h_z^{'} - \rho_0^{'} \frac{v^{'}}{k^{'}} w^{'} + \rho_0^{'} v^{'} (\left[\frac{d}{dx}\right]^2 - k^{'2}) w^{'} - g\,\delta P^{'} + g\,\alpha^{'} \rho_0^{'} \theta^{'} \quad (14)$$

$$m\delta\rho^{'} + \rho_0^{'} w^{'} \frac{d\Omega}{dx} = 0 \quad (15)$$

$$m\theta^{'} - \wedge w^{'} = K_T^{'} (\left[\frac{d}{dx}\right]^2 - K^{'2}) \theta^{'} \quad (16)$$

$$m h_x^{'} = H^{'} \frac{d}{dx} u^{'} + \eta^{'} (\left[\frac{d}{dx}\right]^2 - K^{'2}) h_x^{'} \quad (17)$$

$$m h_y^{'} = H^{'} \frac{d}{dx} v^{'} + \eta^{'} (\left[\frac{d}{dx}\right]^2 - K^{'2}) h_y^{'} \quad (18)$$

$$m h_z^{'} = H^{'} \frac{d}{dx} w^{'} + \eta^{'} (\left[\frac{d}{dx}\right]^2 - K^{'2}) h_z^{'} \quad (19)$$

$$K_x^{'} u^{'} + K_y^{'} v^{'} = i \frac{d}{dx} w^{'} \quad (20)$$

$$K_x^{'} h_x + K_y^{'} h_y^{'} = i \frac{d}{dx} h_z^{'} \quad (21)$$

where, symbol D denotes $\frac{d}{dx}$.

Equations (12) to (21) on simplifying, yield

$$m(\left[\frac{d}{dx}\right]^2 - K^{'2}) w^{'} = \frac{\mu_e^{'} H^{'}}{4\pi \rho_0^{'}} (\left[\frac{d}{dx}\right]^2 - K^{'2}) \frac{d}{dx} h_Z^{'} - \frac{v^{'}}{K^{'}} (\left[\frac{d}{dx}\right]^2 - K^{'2}) w^{'} + v^{'} (\left[\frac{d}{dx}\right]^2 - K^{'2})^2 w^{'} - \frac{gK^{'2} \frac{d\Omega}{dx}}{m} w^{'} - g\alpha^{'} K^{'2} \theta^{'} \quad (22)$$

$$[m - K_T^{'} (\left[\frac{d}{dx}\right]^2 - K^{'2})] \theta^{'} = \wedge w^{'} \quad (23)$$

$$[m - \eta^{'} (\left[\frac{d}{dx}\right]^2 - K^{'2})]\, h_Z^{'} = H^{'} \frac{d}{dx} w^{'} \quad (24)$$

Equations from (22) to (24) give the relevant equations in dimensionless [2] form as:

$$(\left[\frac{d}{dx}\right]^2 - a^{'2})(\left[\frac{d}{dx}\right]^2 - a^{'2} - \sigma^{'} - B^{'}) w^{'} + \frac{\mu_e^{'} H^{'} l}{4\pi \rho_0^{'} v^{'}} (\left[\frac{d}{dx}\right]^2 - a^{'2}) \frac{d}{dx} h_Z^{'} = g \frac{l^4 \left(\frac{dg}{dx}\right) a^{'2}}{\sigma^{'} v^{'2}} w^{'} + \frac{g\alpha^{'} a^{'2} l^2}{v^{'}} \theta \quad (25)$$

$$(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})\theta^{'} = \left(-\frac{\wedge l^2}{K_T^{'}}\right)w^{'} \quad (26)$$

$$(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})h_Z^{'} = \left(-\frac{H^{'}l}{\eta^{'}}\right)\frac{d}{dx}w^{'} \quad (27)$$

The quantities lacking dimensions are $\frac{d}{dx} = l\frac{d}{dx}$, $a^{'} = k^{'}l$, $m = \sigma^{'}v^{'}/l^2$,

$$P_1^{'} = \frac{v^{'}}{K_T^{'}},\ P_2^{'} = \frac{v^{'}}{\eta^{'}}, B^{'} = \frac{l^2}{K^{'}}, Q^{'} = \frac{\mu_e^{'}H^{'2}l^2}{4\pi\rho_0^{'}v^{'}\eta^{'}}, R^{'} = \frac{g\alpha^{'}\wedge l^4}{K_T^{'}v^{'}}$$

$$R_1^{'} = \frac{gl^4(d\Omega/dx)}{k_T^{'}v^{'}},\ F^{'} = \frac{g\alpha^{'}a^{'2}l^2}{v^{'}}\theta^{'} \quad (28)$$

where, $a^{'}$ symbolizes the radians covered per unit of distance, $P_1^{'}$ and $P_2^{'}$ are the variables of fluid under consideration, $Q^{'}$ represents the Chandrasekhar number, $R^{'}$ denotes the Rayleigh number, $R_1^{'}$ is the density stratification number and $B^{'}$ denotes the porosity number. Since the planes are unbounded, so as discussed by Chandrasekhar[3], the boundary conditions become,

$$w^{'} = \left[\frac{d}{dx}\right]^2 w^{'}, \theta^{'} = \frac{d}{dx}h_Z^{'} = 0 \text{ when } x = 0 \text{ and } x = l \quad (29)$$

We have a dispersive relationship from equations (25) to (27) is

$$[(\left[\frac{d}{dx}\right]^2 - a^{'2})(\left[\frac{d}{dx}\right]^2 - a^{'2} - \sigma^{'} - B^{'})(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'}) + R^{'}a^{'2}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'}) - Q^{'}\left[\frac{d}{dx}\right]^2(\left[\frac{d}{dx}\right]^2 - a^{'2})(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1\sigma^{'})$$

$$-\frac{R_1^{'}a^{'2}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma)(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})}{\sigma^{'}P_1^{'}}]w^{'} = 0$$

(30)

Now we express our problem as relation expressing stability in Rayleigh number [2] for the modes showing non oscillatory behaviour. It implies that $\sigma^{'}$ is having real value. We eliminate $h_Z^{'}$ from equations (25) and (27), we have

$$(\left[\frac{d}{dx}\right]^2 - a^{'2})(\left[\frac{d}{dx}\right]^2 - a^{2} - \sigma^{'} - B^{'})(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})w^{'} - Q^{'}(\left[\frac{d}{dx}\right]^2 - a^{'2})\left[\frac{d}{dx}\right]^2 w^{'} -$$

$$\frac{gl^4(d\Omega/dt)}{\sigma^{'}v^{'2}}a^{'2}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})w^{'} = (\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})F^{'} \quad (31)$$

From equation (26), we have

$$(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})F^{'} = -R^{'}a^{'2}w^{'} \quad (32)$$

Multiply the equation (31) by $\overline{w^{'}}$ the complex conjugate of w, on both sides and integrating with respect to x, we get

$$\int_0^l \left\{ \left| \left[\frac{d}{dx}\right]^3 w^{'} \right|^2 + (A^{'}+C^{'}+a^{'2}) \left| \left[\frac{d}{dx}\right]^2 w^{'} \right|^2 + [A^{'}C^{'}+a^{'2}(A^{'}+C^{'})] \left| \frac{d}{dx} w^{'} \right|^2 + a^{'2}A^{'}C^{'} \left| w^{'} \right|^2 \right\} dx \quad (33)$$

$$+Q^{'}\int_0^l \left(\left| \left[\frac{d}{dx}\right]^2 w^{'} \right|^2 + a^{'2} \left| \frac{d}{dx} w^{'} \right|^2 \right) dx + E^{'}\int_0^l \left(\left| \frac{d}{dx} w^{'} \right|^2 + A^{'} \left| w^{'} \right|^2 \right) dx = -\int_0^l \overline{w^{'}} \left(\left[\frac{d}{dx}\right]^2 - A^{'} \right) F^{'}\, dx$$

We take the product of equation (32) with $\overline{F}^{'}$ (where $\overline{F}^{'}$ is the complex conjugate of $F^{'}$) and taking integral within given boundary conditions, we get

$$\int_0^l \left(\left| \frac{d}{dx} F^{'} \right|^2 + J^{'} \left| F^{'} \right|^2 \right) dx = R^{'}a^{'2}\int_0^l \overline{F^{'}} w^{'}\, dx \quad (34)$$

Again multiplying the equation (32) by $((\frac{d}{dx})^2 - A^{'})\overline{F^{'}}$ and integrating, we get

$$\int_0^l \left(\left| \left[\frac{d}{dx}\right]^2 F^{'} \right|^2 + (A^{'}+J^{'}) \left| \frac{d}{dx} F^{'} \right|^2 + A^{'}J^{'} \left| F^{'} \right|^2 \right) dx = -R^{'}a^{'2}\int_0^l w^{'} \left(\left[\frac{d}{dx}\right]^2 - A^{'} \right) \overline{F^{'}}\, dx \quad (35)$$

Here we have notations,

$A^{'} = a^{'2} + P^{'}_2 \sigma^{'}$,

$C^{'} = a^{2} + \sigma^{'} + B^{'}$,

$E^{'} = gl^4 (d\Omega/dx) a^{'2} / \sigma^{'} v^{'2}$,

$J^{'} = a^{'2} + P_1^{'} \sigma^{'}$

using above equations, we have,

$$R^{'} = \frac{1}{a^{'2}} \left(\frac{I_1^{\zeta}}{I_2^{\zeta}} \right) \quad (36)$$

where,

$$I_1^{\zeta} = \int_0^l (\left| \left[\frac{d}{dx}\right]^2 F^{'} \right|^2 + (A^{'}+J^{'}) \left| \frac{d}{dx} F^{'} \right|^2 + A^{'}J^{'} \left| F^{'} \right|^2) dx$$

$$I_2^{\zeta} = \int_0^l (\left| \left[\frac{d}{dx}\right]^2 w^{'} \right|^2 + (A^{'}+C^{'}+a^{'2}+Q^{'}) \left| \left[\frac{d}{dx}\right]^2 w^{'} \right|^2 + (A^{'}C^{'}+a^{'2}A^{'}+a^{'2}C^{'}+Q^{'}a^{'2}+E^{'}) \left| \frac{d}{dx} w^{'} \right|^2 + (a^{'2}A^{'}C^{'}+E^{'}A^{'}) \left| w^{'} \right|^2) dx$$

Let $\delta R^{'}$ be a corresponding small change in $R^{'}$ as $w^{'}$ and $F^{'}$ change to δw and $\delta F^{'}$ respectively, consistent with boundary conditions,

$\delta w^{'} = \delta F^{'} = 0$ when $x = 0$, $x = l$

and $\left[\frac{d}{dx}\right]^{2q} (\delta w^{'}) = 0$ when $x = 0$, $x = l$ (37)

using above, we have

$$\delta R^{'} = \frac{1}{a^{'2} I_2^{'}} (\delta I_1^{\zeta} - R^{'} a^{'2} \delta I_2^{\zeta}) \quad (38)$$

If δI_1^{ζ} and δI_2^{ζ} denote small changes in $I_1^{'}$ and $I_2^{'}$ respectively, then.

$$\delta I_1^{\zeta} = \int_0^l [(\left[\frac{d}{dx}\right]^2 F^{'}\left[\frac{d}{dx}\right]^2 \overline{\delta F^{'}} + \overline{\left[\frac{d}{dx}\right]^2 F^{'}}\left[\frac{d}{dx}\right]^2 \delta F^{'}) + (A^{'}+J^{'})(\frac{d}{dx}F^{'}\frac{d}{dx}\overline{\delta F^{'}} + \overline{\frac{d}{dx}F^{'}}\frac{d}{dx}\delta F^{'}) + A^{'}J^{'}(F^{'}\overline{\delta F^{'}} + \overline{F^{'}}\delta F^{'})]dx$$

$$\delta I_2^{\zeta} = \int_0^l (\overline{\delta F^{'}}(\left[\frac{d}{dx}\right]^4 - (A^{'}+J^{'})\left[\frac{d}{dx}\right]^2 + A^{'}J^{'})F^{'}dz + \int_0^l \delta F^{'}(\left[\frac{d}{dx}\right]^4 - (A^{'}+J^{'})\left[\frac{d}{dx}\right]^2 + A^{'}J^{'})\overline{F^{'}}dx$$

$$\delta F_2^{'} = \int_0^l -\left\{\overline{\delta w^{'}}\ [\left[\frac{d}{dx}\right]^6 - (A^{'}+C^{'}+a^{2}+Q^{'})\left[\frac{d}{dx}\right]^4 + (A^{'}C^{'}+a^{'2}A^{'}+a^{'2}C^{'}+Q^{'}a^{2}+E^{'})\left[\frac{d}{dx}\right]^2 - (a^{'2}A^{'}+E^{'}A^{'})]w^{'} + \right.$$

$$\left.\int_0^l \delta w^{'}[\left[\frac{d}{dx}\right]^6 - (A^{'}+C^{'}+a^{2}+Q^{'})\left[\frac{d}{dx}\right]^4 + (A^{'}C^{'}+a^{'2}A^{'}+a^{'2}C\ +Q^{'}a^{'2}+E^{'})\left[\frac{d}{dx}\right]^2 - (a^{'2}A^{'}+E^{'}A^{'})]\overline{w^{'}}\right\}$$

From equation (38), we have

$$\delta R^{'} = \frac{1}{a^{'2}I_2^{\zeta}}(P^{'} + \overline{P^{'}} + R^{'}a^{'2}(Q^{'} + \overline{Q^{'}})) \qquad (39)$$

Where:

$$P^{'} = \int_0^l \delta\overline{F^{'}}(\left[\frac{d}{dx}\right]^4 - (A^{'}+J^{'})\left[\frac{d}{dx}\right]^2 + A^{'}J^{'})F^{'}dx$$

$$Q^{'} = \int_0^l \delta\overline{w^{'}}(\left[\frac{d}{dx}\right]^6 - (A^{'}+C^{'}+Q^{'}+a^{'2})\left[\frac{d}{dx}\right]^4 + (A^{'}C^{'}+a^{'2}A^{'}+a^{'2}C^{'}+Q^{'}a^{'2}+E^{'})\left[\frac{d}{dx}\right]^2 - (a^{'2}A^{'}+E^{'}A^{'})]w^{'}$$

or

$Q^{'} = \int_0^l w^{'}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})\delta\overline{F^{'}}\,dx$, with the help of (33)

Again from equation (39),

$$\delta R^{'} = \frac{1}{a^{'2}I_2^{\zeta}}R^{'}[P^{'} + R^{'}a^{'2}Q^{'}]$$

$$= \frac{1}{a^{'2}I_2^{\zeta}}[\int_0^l [\delta\overline{F^{'}}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})F^{'} + R^{'}a^{'2}w^{'}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})\delta\overline{F^{'}}]dx \qquad (40)$$

Conclusion

From (40),we have, when $\delta R^{'} = 0$

then, $\int_0^l [\delta\overline{F^{'}}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})F^{'} + R^{'}a^{'2}w^{'}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})\delta\overline{F^{'}}]dx = 0$

implies.

$$\delta\overline{F^{'}}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})F^{'} = -R^{'}a^{'2}w^{'}(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_2^{'}\sigma^{'})\delta\overline{F^{'}}$$

Implies $\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'} = -R^{'}a^{'2}w^{'}$

So, in this paper, we derived mathematically the impact of different perturbed quantities like velocity components, density, pressure, temperature and the magnetizing field respectively on fluid flow. It follows from above that when $(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})F^{'} = -R^{'}a^{'2}w^{'}$ then we have $\delta R^{'} = 0$.Also the relation holds conversely, equation (40) gives $(\left[\frac{d}{dx}\right]^2 - a^{'2} - P_1^{'}\sigma^{'})F^{'} = -R^{'}a^{'2}w^{'}$ for any arbitrary change $\delta F^{'}$ in $F^{'}$ consistent with the boundary conditions of the problem, moreover the value of $F^{'}$ satisfies the boundary value problem and $R^{'}$ is expressed in terms of $F^{'}$. So, the perturbation problem is solved using calculus of variations.

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