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# PERFORMANCE ANALYSIS OF BACKSTEPPING AND OPTIMIZED FOPID CONTROLLER FOR TRAJECTORY TRACKING OF WHEELED MOBILE ROBOT

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## Abstract

Differential drive mobile robots are now used for a wider range of commercial and industrial applications than just for scientific study. The field of mobile robotics faces a serious challenge with trajectory tracking. The trajectory tracking of the differential-drive wheeled mobile robot is studied in this thesis using a control strategy that combines kinematics based backstepping with optimized fractional-order PID controllers for the dynamics Moreover, to obtain an optimal control system, fuzzy inference system and genetic algorithm optimization techniques are applied separately to tune the parameters of the fractional order PID dynamic controllers. Finally, several simulations are implemented to the trajectory tracking of mobile robots in the cases with and without disturbance signal, and the results can confirm the effectiveness and superiority of the combined control scheme. The fuzzy based FOPID controller response performs better with ITAE of 0.0289 and 0.0364 in the x and y direction whereas with GA based FOPID the ITAE is 0.374 and 0.285 respectively. The robustness of the backstepping controller is also clearly proved.

**Keywords:** *Wheeled Mobile Robot, Backstepping controller, FOPID, Genetic algorithm, Fuzzy Inference system, trajectory tracking*

## 1. Introduction

Mobile robots are among the widely used robots these days. They are not confined to a single physical area and have the ability to move around their surroundings. They are capable of being "autonomous" (AMR, autonomous mobile robot), which implies they can move around freely without the aid of mechanical or electrical guiding systems. Mobile robots include land or home robots (wheeled and legged), aerial robots (unmanned aerial vehicles), and underwater robots (autonomous underwater vehicles)[1].

WMR are frequently employed in industries as 'self-guided' or 'autonomous guided' vehicles. Material handling systems or load carriers known as automated guided vehicles (AGVs) can navigate through a warehouse, distribution center, or production plant without a driver or operator on board. They are frequently used to transport unfinished goods like paper, rubber, metal, or plastic. AGVs, for instance, can deliver materials directly to production lines, transfer raw materials to or from the warehouse, or both. AGVs continuously and dependably supply the necessary raw materials without human intervention, guaranteeing that production lines never run out of the supplies they require[2].

Today's robotic applications place great expectations on productivity and accuracy, which has emphasized a desire to replace traditional control tactics with more modern (intelligent) control approaches[3]. Intelligent control is a type of regulation that mimics human intelligence in problem-solving, learning, and decision-making[4].

As in [5], the implementation of control laws that direct an element to approach and follow a geometric path is the primary focus of path following problems. A secondary objective is to compel the object to meet extra dynamic requirements as it moves along the path.

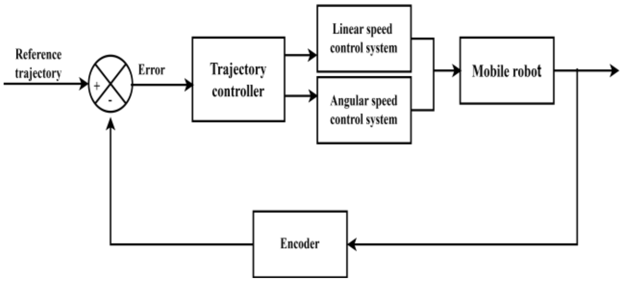

Figure 1 Trajectory Control System Block Diagram [6]

## 2. Related Works

In [7], Zangina, et al studied a simple solution for the path tracking problem of a mobile robot using a PID controller. The method adopted was a trial-and-error technique with six tuning parameters for the robot to track a desired trajectory. The controller was used to overcome the non-linearity of the reference trajectory tracking as well as the speed of the DC motor adjustments. However, using a traditional PID controller can result in a variety of issues, including high overshoot, oscillation of speed and torque due to parameter variation, a slow response rate, always selecting the correct PID gains, and using fixed gains.

Moudoud, Aissaoui et al, presented a robust adaptive trajectory tracking controller for an electrically wheeled mobile robot in the presence of dynamic disturbances [8]. They develop a sliding mode control scheme to ensure robustness by compensating for and weakening the effects of dynamic disturbance. They also synthesized an improved adaptive switching control capable of mitigating the chattering phenomenon and optimizing the convergence rate. A back-stepping control approach was adopted for the kinematics of the system. However, their attempts to avoid chattering were not enough.

According to Hassani et al. [10], a back-stepping approach was used to achieve the path tracking of the nonholonomic WMR. The back-stepping control was designed for the kinematics of nonholonomic WMR. In order to improve the performance of the back-stepping controller and minimize the errors obtained by this system, a genetic algorithm was applied. But this paper doesn’t account for the effect of the dynamics of the system.

In[11] they proposed a novel adaptive sliding mode controller (ASMC) combined with a fuzzy PID algorithm that leads to global asymptotic stability. The sliding mode approach effectively solved the problem of uncertainties in the dynamics of the system; however, the chattering phenomenon was the main drawback of their method. An improved adaptive and robust fuzzy PID controller was presented as a three-layer controller that can be considered for tracking and regulation tasks. Despite their relatively improved results, they didn’t try to compare their results with other optional intelligent optimization techniques.

In [12], the authors discuss the adaptive sliding mode trajectory tracking control for wheeled mobile robots (WMR) in the presence of external disturbances and inertia uncertainties. A new fast non-singular terminal sliding mode surface without any constraints was proposed, which not only avoided the singularity, but also retained the advantages of sliding mode control. In order to implement the trajectory tracking mission, the error dynamic system was divided into a second-order subsystem and a third-order. First, an adaptive fast non-singular terminal sliding mode control law of angular velocity was developed for finite-time stabilization of the second-order subsystem. Then, another adaptive fast non-singular terminal sliding mode control law of the linear velocity was designed to guarantee the stability of the third-order subsystem.

An integral adaptive low-based robust sliding mode controller was proposed by Azzabi and Nouri [13] to compensate for unknown external disturbances. Adaptive estimation is performed for each perturbation of each subsystem separately bounded by an unknown local upper bound; a known upper bound of the disturbance is not required. Despite their robustness, sliding mode controllers suffer from chattering phenomena.

According to [14], FOPID is gaining popularity these days with the improvement in its results in comparison to the classical one. They proposed a controlling action for a nonlinear system using a FOPID controller in their work. The optimal tuning method (GA) was adopted to get the best parameter of the controller. The tendency in optimizing the parameters that has been adopted here is the application of GA, which is a powerful search technique in the world of optimization. They have used ITAE performance index as a fitness function in optimization techniques using GA. From their simulation results, it clearly depicts that the GA has an obvious effect on improving the performance of the FOPID controller. However, they didn’t try to compare its performance with other optimization methods.

In[15], they exploited the advanced features of the FO controller as an adjustable speed derivation of PMDM. The dominant parameters of the proposed controller were tuned using the particle swarm optimization technique, and the result was compared with that of a traditional integer-order PID controller. The ITAE fitness function was used as a performance criterion for the output response of the system. Despite the system's nonlinear behavior, their results demonstrated accurate speed tracking for no load and full load conditions. The performance shows a superior dynamic response. However, no attempt was made to compare the PSO method with other powerful optimization techniques.

In [16], The fractional order PIDF fuzzy controller (Fuzzy-FOPIDF), the PID controller, and the fractional order PID controller were compared. The Kinematic Controller was based on inverse kinematics, and the Dynamic Controller was a fuzzy fractional order PID controller. In order to ensure better trajectory following, they have developed a law made up of two loops nested one inside the other. The internal loop was representing the dynamic part (fuzzy FOPID) of the robot, and the external loop was representing the kinematic part of the robot. When compared to fuzzy PID controllers, the fuzzy FOPID controller achieves a much lower trajectory tracking error with a much shorter response time. But it would be better if they compared the effect of the fuzzy with other optimization methods.

## 3. Method

### 3.1 Robot kinematic modeling

The primary objective of kinematic modelling for the DDMR is to depict the robot velocity as a function of the velocity of the driving wheels and the robot's geometric parameters.

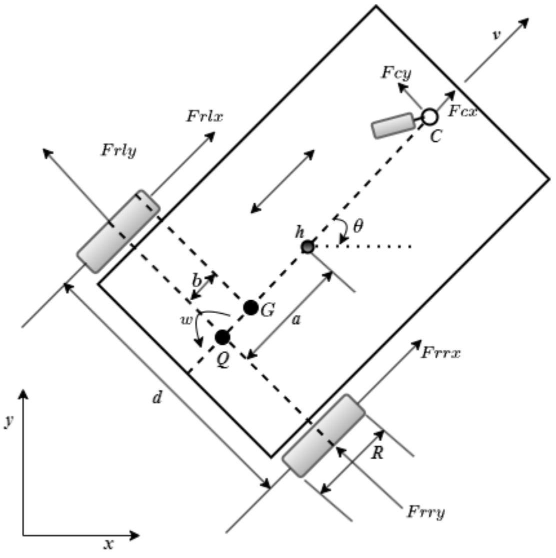

Figure 2 Differential derived mobile robot schematic diagram[17].

The components of the model shown in the diagram are as follows: $G$, the mass center of the moving robot; $h$, the simplified point of the moving robot in the x-y plane; $\theta$, the directional angle of the moving robot; $a$, the distance between $h$ and $B$, the axis centre of the wheels; $b$, the distance between $G$ and $B$; $d$, the distance between the two rear wheels; and $C$, a caster to balance the moving robot.

The DDMR velocities can be described also in the inertial frame as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -a\sin\theta \\ \sin\theta & a\cos\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{1}$$

3.2 Robot dynamic modeling

The construction of various motion control strategies and the simulation analysis of the autonomous vehicle's motion both require a dynamic model of the vehicle.

$$\begin{aligned} \sum Fx &= Frlx + Frrx + Fex + Fc \\ &= m(\dot{u} - \bar{u}\omega) \end{aligned} \tag{2}$$

$$\begin{aligned} \sum Fy &= Frly + Frry + Fey + Fc \\ &= m(\dot{\bar{u}} - u\omega) \end{aligned} \tag{3}$$

$$\begin{aligned} \sum Mz &= Iz\dot{\omega} = \frac{d}{2}(Frrx - Frlx) - b(Frl \\ &\quad - Frry) + (e - b)Fey + (c \\ &\quad - b)Fcy + \tau e \end{aligned} \tag{4}$$

where $Iz$ and $m$ are the inertia moment of the robot on the vertical axis and mass of the robot.

The majority of robots on the market today, come equipped with low-level PID velocity controllers to monitor input reference velocities. Therefore, to properly characterize the mobile robot model, it is useful to include both linear and angular reference velocities as control signals[18]. For this reason, the velocity controls are included in the model. This velocity will often be referred to as "linear velocity" because point B only has a longitudinal velocity (u) in the absence of slip. PD velocity controllers, which have been considered to simplify the model, are described by the following equation:

$$\begin{bmatrix} v_u \\ v_\omega \end{bmatrix} = \begin{bmatrix} k_{PT}\left(u_{ref} - u_{me}\right) - k_{DT}\dot{u}_{me} \\ k_{PR}\left(\omega_{ref} - \omega_{me}\right) - k_{DR}\dot{\omega}_{me} \end{bmatrix} \tag{5}$$

Where:

$$\boldsymbol{u_{me}} = \frac{\mathbf{1}}{\mathbf{2}}[\boldsymbol{r}(\boldsymbol{\omega r} + \boldsymbol{\omega l})]$$

$$\omega_{me} = \frac{1}{d}[r(\omega r - \omega l)]$$

$$v_u = \frac{vl + vr}{2}$$

$$v_\omega = \frac{vl - vr}{2}$$

$$\begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\theta_3}{\theta_1}\omega^2 - \frac{\theta_4}{\theta_1}v \\ -\frac{\theta_5}{\theta_2}v\omega - \frac{\theta_6}{\theta_2}\omega \end{bmatrix} + \begin{bmatrix} \frac{1}{\theta_1} & 0 \\ 0 & \frac{1}{\theta_2} \end{bmatrix} \begin{bmatrix} u_v \\ u_w \end{bmatrix} + \begin{bmatrix} \delta_v \\ \delta_w \end{bmatrix} \quad (6) \tag{0.1}$$

Where, $\delta_v$, $\delta_\omega$ are the variables indicating the uncertainty of $v$ and $\omega$; $u_v$ and $u_\omega$, the reference velocities, can be taken as the control inputs of the mobile robot.

## 4. Control design

### 4.1 Kinematic Controllers

In [19] they provided a tracking control algorithm for a nonholonomic mobile robot that is based on the steering system and ignores vehicle dynamics. The reference posture and the current posture of the robot will both be used in this control system. The reference posture is the desired posture, while the actual posture is the one, you're in, right now. The kinematic-based controller is unable to generate a tracking response that is desirable for two different reasons. For the first reason, the system dynamics, including the mass, moment of inertia, and Coriolis and centrifugal velocities of the vehicles, are ignored. The second reason is that one reference trajectory and the robot's starting position should be used to calibrate the controller gains[20]. For one reference trajectory, one set of gains will work and another set would not.

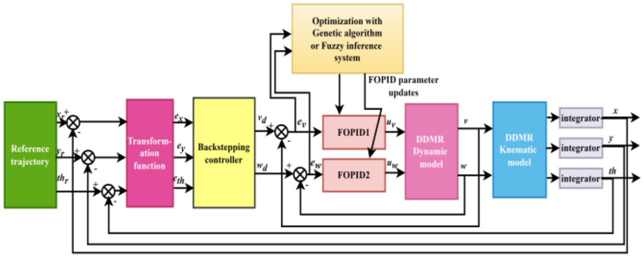

Figure 3 Control system for the trajectory tracking of DDMR.

In this situation, the control task will be to find a control rule for the vehicle that calculates the desired velocity $v_c = f\left(e_p, v_r, K\right)$ and keeps the system asymptotically stable. The following is the proposed kinematic-based control rule:

$$v_c = \begin{bmatrix} v_r \cos e_\theta + K_x e_x \\ \omega_r + K_y v_r e_y + K_\theta v_r \sin e_\theta \end{bmatrix} \tag{7}$$

Where, a $K_x, K_y, K_\theta$ are positive constants.

The stability of the above control rule will be proved using the Lyapunov stability method as follow.

$$\dot{e}_p = \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_\theta \end{bmatrix} = \begin{bmatrix} \omega e_y - v + v_r \cos e_\theta \\ -\omega e_x + v_r \sin e_\theta \\ \omega_r - \omega \end{bmatrix} \tag{8}$$

## 4.2 Design of the FOPID controller

We employ a fractional-order PID controller for the dynamic control of the mobile robot, which is given by equation:

$$u(t) = k_p e(t) + k_i \frac{d^{-\lambda}}{dt^{-\lambda}} e(t) + k_d \frac{d^{\mu}}{dt^{\mu}} e(t) \tag{9}$$

Where Its Laplace transform is as follows

$$c(s) = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu \tag{10}$$

where $u = [u v, u\omega]$ is the dynamic controller's output, $e = [e v, e\omega]$ is the error for the mobile robot's linear and angular velocities. Figure 4.4 depicts the architecture of the FOPID controller, and two FOPID controllers are used to adjust the velocities of the mobile robot.

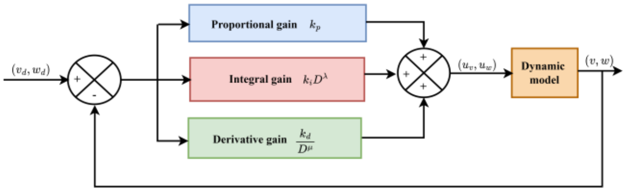

Figure 4 Dynamic controller of DDMR based on FOPID.

## Fuzzy FOPID:

Two inputs and five outputs comprise the FLC in this study. The outputs are the fractional order PID controller parameter gains, with errors and differences in error serving as inputs. The discrepancy between the target or reference value and the output of the plant is the error.

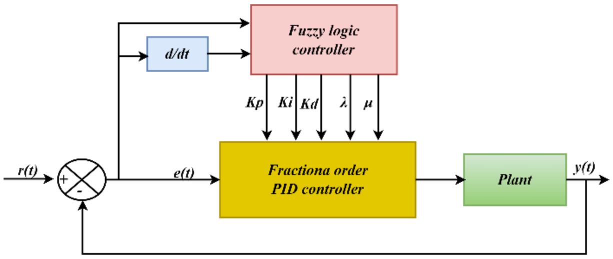

Figure 5 Fuzzy based Fractional order PID controller

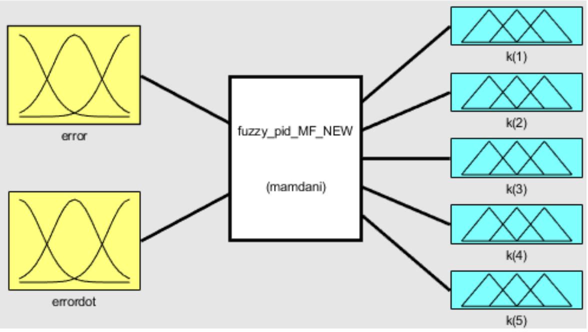

Figure 6 Fuzzy inference system for tuning the FOPID parameters

## GA based FOPID

GA is a stochastic search technique that has been applied to the solution of challenging optimization issues[21]. Natural selection, recombination, and chromosomal mutation are the three basic GA components. The Darwinian principles of evolution and the "survival of the fittest" are the foundation of GA to come up with an optimal solution. The population changes with each iteration to produce a fitter group of individuals[22, 23].

The following flowchart shows the basic step of genetic algorithm.

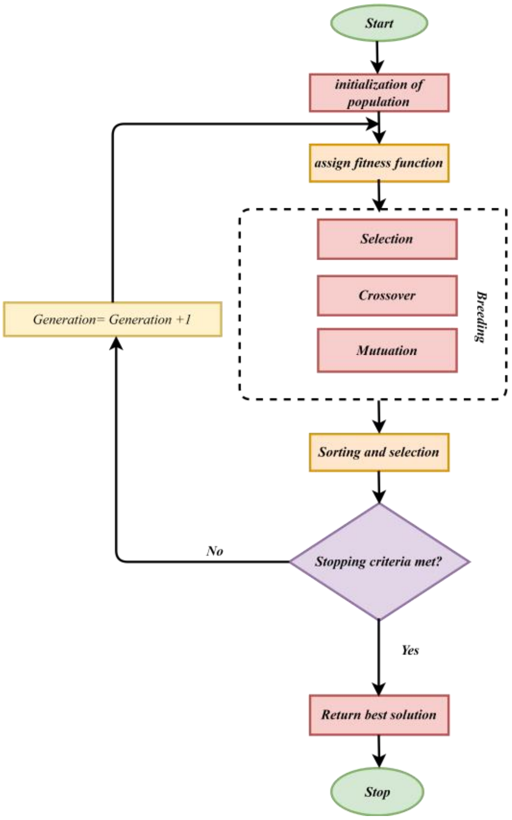

Figure 7 Flow diagram of genetic algorithm

## 5. Simulation and result

The overall Simulink model for the trajectory tracking control of the DDMR is presented as in the preceding subtopics. The wheeled mobile robot is represented kinematically and dynamically in the Simulink model, together with function blocks for trajectory generation, a backstepping speed controller for supplying velocity commands, and an optimized fractional order PID dynamic controller.

The genetic algorithm is used to determine the best set of parameters based on the given fitness function.

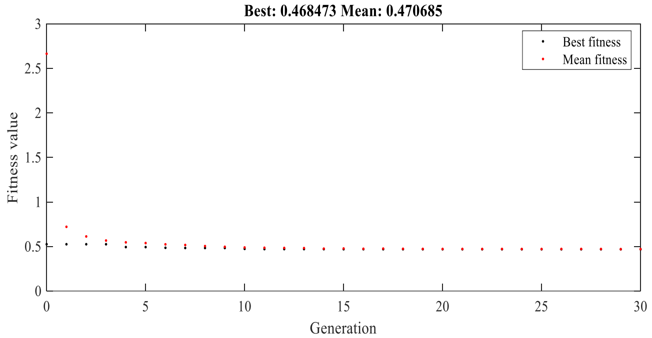

Figure 8 Best and mean fitness of the GA based solution

The output response obtained with the fuzzy inference based FOPID controller for the dynamics of the system clearly reveals that the controller tracks the set point with a very small rising and settling time but with a little overshoot. The steady state error is also insignificantly small.

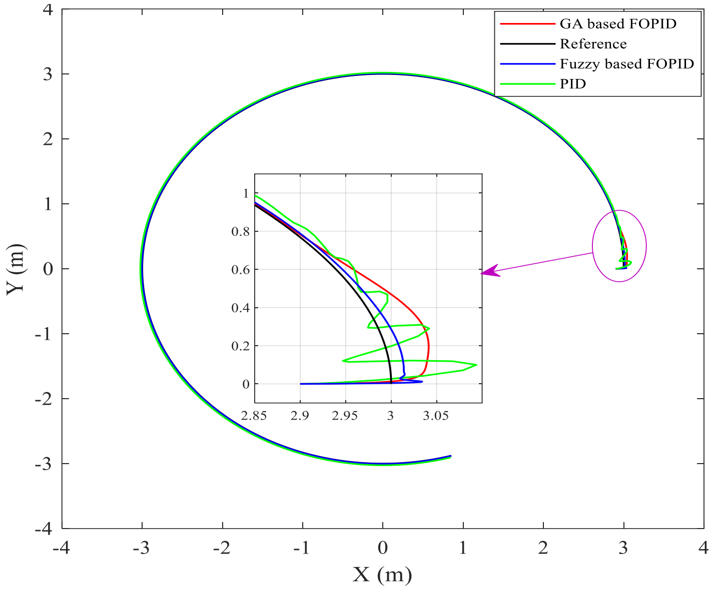

Figure 9 circular trajectory response using different controllers

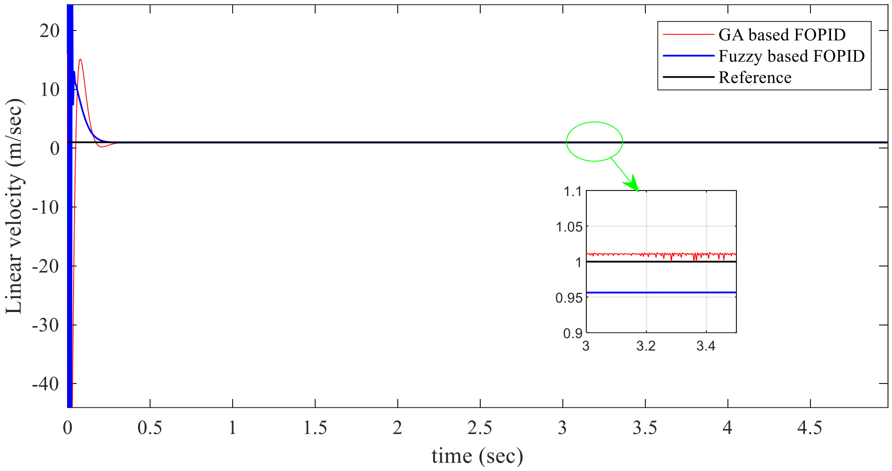

Figure 10 Linear velocity response using the proposed controllers

The linear and angular velocity response of the system using fuzzy inference based FOPID and GA based FOPID controllers are also presented as shown above in figure 12. with fuzzy inference based FOPID the linear velocity response shows that the proposed controller has effective performance with very small settling time and a small steady state error for the dynamics of the wheeled mobile robot despite the high oscillation near to the origin. The linear velocity response with GA based FOPID controller has better steady state error. But there is a chattering phenomenon, which cannot be neglected.

The angular velocity response in contrast has a notable steady state error with GA based FOPID controller than fuzzy based FOPID controller. The settling time for both controllers is small and comparable to each other. The chattering phenomenon is still unavoidable in the GA based FOPID controller response.

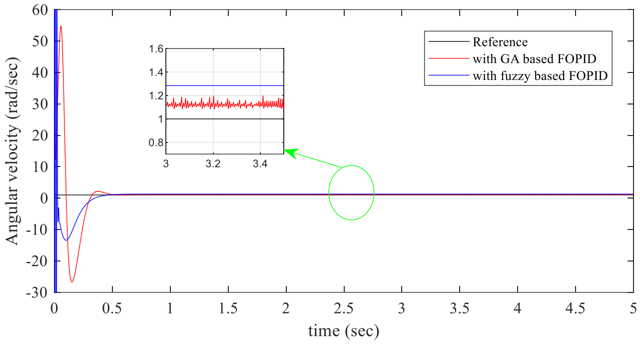

Figure 11 Angular velocity response with the proposed controllers.

The ability of the controller to reject undesired disturbances is assessed by injecting a perturbation signal, as illustrated in Figure 5.9 below, with a magnitude of 0.5 at time 3 sec, after a settling time is reached. The disturbance signal is added to the control input of the system's dynamic model because the dynamics of the system are what cause the majority of disturbances.

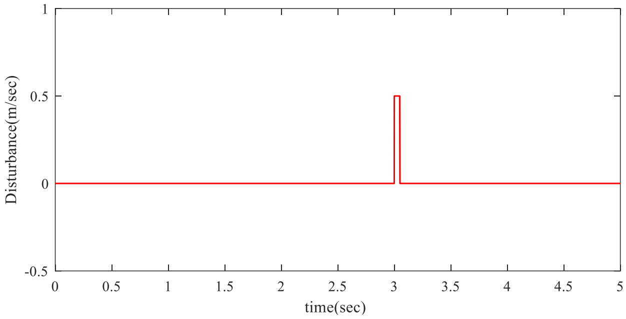

Figure 12 Disturbance signal

It performs well despite the applied disturbance signal. So, the proposed controller integration has excellent performance in tracking the reference trajectory of the mobile robot.

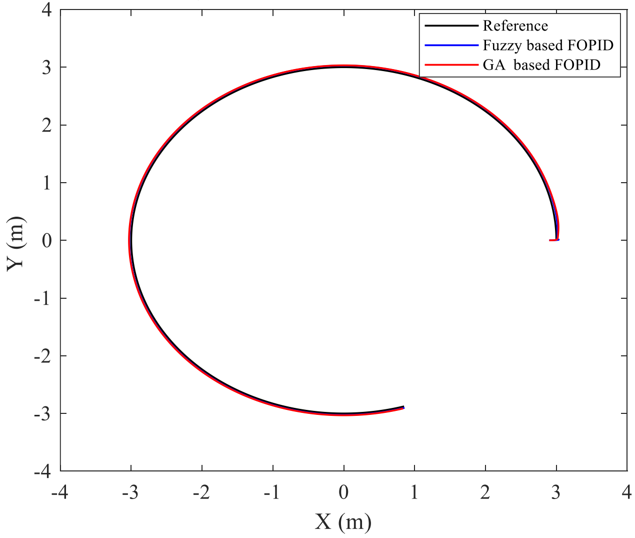

Figure 13 Circular trajectory tracking response with disturbance signal

The figures show that there are some undesirable results in the linear and angular response after the disturbance signal is applied. There are noticeable overshoots at the start of the disturbance, undershoot at the end of the disturbance, and it takes some time to settle again. This clearly demonstrates that the proposed controllers for the dynamic model are not good enough for rejecting the applied disturbance signal and the robustness of the proposed controller emanates from the kinematic-based backstepping controller. The kinematic based backstepping controller performs well in attenuating the disturbance signal.

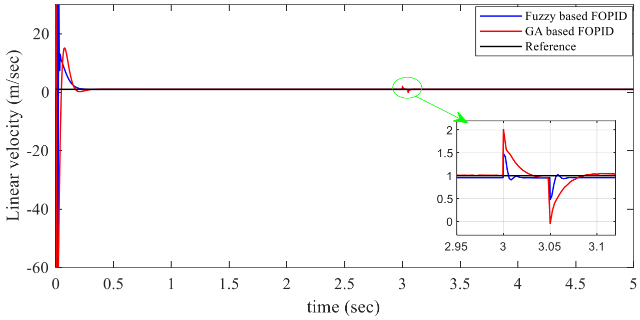

Figure 14 Linear velocity control input when disturbance signal is applied

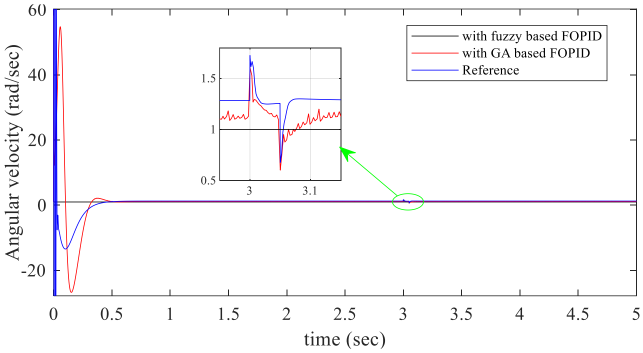

Figure 15 Angular velocity control input disturbance signal is applied

## 6. Conclusion and Future work

The trajectory tracking problem of a nonholonomic wheeled mobile robot is addressed in this paper using a combination of kinematic-based backstepping and fractional order PID controllers. The inner loop (system dynamics) of the proposed controller uses a fractional order PID (FOPID), and the outer loop (kinematics of the system) uses a backstepping controller. Different optimization techniques are used in the earlier controller to obtain the $kp, ki\ kd\ \ \lambda\ and$ μ parameters of the FOPID controller.

Fuzzy logic and genetic algorithm optimization techniques are applied for optimal tuning of the controllers' parameters, with ITAE used as an objective function. The proposed control approaches were compared to one another and to traditional tuning approaches. The performance of the controller was analyzed using performance measures (time domain specification). The proposed controller's performance was determined through simulation using the MATLAB 2021a software. The results show that the fuzzy inference-based tuning of the FOPID controller parameters has better performance on the overall trajectory response of the robot than the GA-based FOPID controller tuning method.

The kinematic-based backstepping controller works excellently at following the desired trajectory with the minimum amount of error, and it exhibits great disturbance rejection (robustness) performance, which the proposed dynamic controllers are not good at.

As can be observed from the simulation results, kinematic-based backstepping controllers with fuzzy inference-optimized FOPID controllers offer higher performance characteristics in terms of trajectory error and robustness against perturbation than the other strategies covered in this work. In order to verify simulation results and conduct real-time implementation, it is advised to carry out practical implementation of these controllers for trajectory tracking control of the wheeled mobile robot. This is due to the possibility that there could be modelling inaccuracies when creating the robot, meaning that a strategy that worked best in the model could not always perform effectively in reality.

Furthermore, the established control strategy and optimization method may be applied to a variety of applications, including robot manipulators, unmanned aerial vehicles, and others, by simply altering the plant system.

## Code and data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

## Conflicts of Interest

No conflict of Interest

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